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Resonances in the Stark Effect of Atomic Systems

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Abstract. Generalizing earlier results on the Hydrogen case it is proved, through a dilation analyticity technique different from the canonical one, that the action of a weak electric field shifts the isolated eigenvalues of any atomic system into resonances of the Stark effect, uniquely determined by the perturbation series through the Borel summation method.

I. Introduction

This paper represents a continuation of a preceding one [3] in which the existence of resonances in the Hydrogen Stark effect and the Borel summability of the divergent perturbation series were proved through the well known separability of the problem in squared parabolic coordinates. These results have been independently and almost simultaneously obtained by Herbst [5] for a two-body Stark operator with a dilation analytic potential of general type. Furthermore Herbst and Simon [6] have announced a generalization [7]; see also Hunziker [8] of Herbst's results to a N-body Schrödinger operator of the type

$$H(F) = -\sum_1^N \Delta_i - \sum_1^N V_i(\mathbf{r}_i) + \sum_{i<k}^N V_{ik}(\mathbf{r}_i - \mathbf{r}_k) + \sum_1^N z_i \tag{1.1}$$

which includes the operator describing the Stark effect on any atomic system obtained for $V(\mathbf{r}_i) = Z/|\mathbf{r}_i|$, $V_{ik} = 1/|\mathbf{r}_i - \mathbf{r}_k|$, if Z is the atomic number, F the electric field strength, and the electron charge is put equal to one.

The key argument of [5] is the discovery that the canonically dilated operator $-e^{-2\phi}\Delta + Fe^{\phi}z$ associated with $-\Delta + Fz$ is, when defined on $D(-\Delta) \cap D(z)$, a holomorphic family of type A with empty spectrum for $0 < |\mathrm{Im}\,\phi| < \pi/3$. This allows to extend the dilation analyticity technique [1, 2] to the Stark problem, which is not dilation analytic according to the usual notion of this concept (see e.g. Reed and Simon [10, Sect. XIII.10]).

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