# On Edwards' Model for Polymer Chains:

## II. The Self-Consistent Potential

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**Abstract.** We obtain an existence and uniqueness theorem for the self-consistent potential in Edwards' model for polymer chains, and confirm the asymptotic analysis proposed by him on the basis of WKB arguments.

## Introduction

The present paper is a reprise of the original work of Edwards' [1] on his continuum model for long polymer chains, the object being to place the results stated there on a firm basis. In [1] Edwards proposes as an approximation to the polymer model a Markov process. This process is a drift process characterised by a spherically symmetric non-negative potential function which is to satisfy a non-linear equation whose structure is motivated by the polymer model (self-consistency condition). We prove existence and uniqueness of this self-consistent potential, and confirm that its asymptotic behaviour is that proposed by Edwards. The proof relies on the fact that the self-consistent potential must satisfy a certain non-linear differential equation. This equation is studied in Sect. 1, its relation to the polymer problem being given in Sect. 2. In Sect. 3 we prove a limit theorem for drift processes from which Edwards' main conclusion concerning the predictions of the Markovian model for the length of polymer chains follows.

In this paper no attempt is made to prove that the Markovian model is a sufficiently good approximation to the original polymer chain model that the limit theorem proved in Sect. 3 for the Markovian model applies to the original. Indeed the existence theorem for the polymer model we have obtained in [2] does not provide a sufficient basis for making such an attempt. The theorem proved in [2] asserts that the polymer measure is well-defined on paths parametrised by the time interval $[0, 1]$ for sufficiently small coupling constant. For a fixed coupling constant $g$ this is equivalent to the assertion that the measure is well-defined on paths parametrised by $t\varepsilon[0, T]$ for $T = T(g)$ sufficiently small. This restriction must be removed if the limiting behaviour of paths as $t \to \infty$ is to be considered. We remark, however, that some problems of this kind have been solved by application of the Donsker–Varadhan theory of the asymptotics of functionals of Markov processes [3].