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# Vlasov Hydrodynamics of a Quantum Mechanical Model

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**Abstract.** We derive the Vlasov hydrodynamics from the microscopic equations of a quantum mechanical model, which simulates that of an assembly of gravitating particles. In addition we show that the local microscopic dynamics of the model corresponds, on a suitable time-scale, to that of an ideal Fermi gas.

## 1. Introduction

It is only in certain limited contexts (e.g. [1–3]) that the extraction of macroscopic dynamical laws from the microscopic equations of motion of 'large' quantum systems has been rigorously effected. To the best of our knowledge, there are, as yet, no rigorous derivations of hydrodynamics from quantum statistical mechanics.

The present article is devoted to the passage from the microscopic equations of motion to a form of hydrodynamics, i.e. that due to Vlasov, for a quantum mechanical model which, though relatively simple, does have some physical significance, as we shall presently explain. The model, $\Sigma^{(N)}$, is that of an assembly of $N$ particles of the same species, in $\mathbb{R}^3$, with Hamiltonian

$$H^{(N)} = \frac{1}{2} N^{-2/3} \sum_{j=1}^{N} p_j^2 + N^{-1} \sum_{\substack{j,k=1 \\ j<k}}^{N} V(x_j - x_k), \tag{1.1}$$

where

$$[x_j, p_k] = i\delta_{jk}, [x_j, x_k] = [p_j, p_k] = 0, \tag{1.2}$$

and where $V$ satisfies certain regularity conditions. As we are concerned with the properties of the model in the limit $N \to \infty$, we formulate the dynamics of a sequence of systems $\{\Sigma^{(N)}\}$. Furthermore we restrict our considerations, for

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