Commun. Math. Phys. 78, 545–566 (1981)

Communications in Mathematical Physics

Lattice Systems with a Continuous Symmetry

III. Low Temperature Asymptotic Expansion for the Plane Rotator Model

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Abstract. We prove that the expansion in powers of the temperature T of the correlation functions and the free energy of the plane rotator model on a d-dimensional lattice is asymptotic to all orders in T. The leading term in the expansion is the spin wave approximation and the higher powers are obtained by the usual perturbation series. We also prove the inverse power decay of the pair correlation at low temperatures for $d=3$.

I. Introduction

We investigate the low temperature properties of the classical plane rotator model described by the Hamiltonian:

$$-\beta H=\beta \sum_{\langle ij\rangle} \cos(\phi_i-\phi_j), \qquad \phi_i\in[-\pi,\pi], \tag{1}$$

β is the inverse temperature T and $\langle i,j\rangle$ are nearest neighbor pairs of sites on the d-dimensional simple cubic lattice Z^d.

It has been known for a long time that the SO(2) symmetry of this model is only broken in $d\geqq 3$ where there is a spontaneous magnetization at low temperatures [8, 13]. These results provide a qualitative justification of the spin wave picture. In this paper we prove that in any dimension d, the free energy and the correlations have a low temperature expansion about the spin wave approximation valid to all orders in T. In particular we show how to get higher order correction in T to the spontaneous magnetization ($d\geqq 3$). The zeroth order value for the spontaneous magnetization was obtained in [13].

* Supported by NSF Grant No. MCS 78-01885
** Supported by NSF Grant No. PHY 78-15920
*** Supported by NSF Grant No. DMR 73-04355
+ Supported by NSF Grant No. PHY-7825390 A01
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0010-3616/81/0078/0545/$04.40