Commun. Math. Phys. 78, 507–530 (1981)

Communications in Mathematical Physics

A Mechanical Model of Brownian Motion

D. Dürr*, S. Goldstein**, and J. L. Lebowitz***

Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

Abstract. We consider a dynamical system consisting of one large massive particle and an infinite number of light point particles. We prove that the motion of the massive particle is, in a suitable limit, described by the Ornstein-Uhlenbeck process. This extends to three dimensions previous results by Holley in one dimension.

0. Introduction

The erratic motion of a macroscopically small but microscopically large particle (visible in a microscope), in an equilibrium fluid, e.g. pollen in a liquid or a smoke particle in air, is a well known physical phenomenon going under the name of Brownian motion [1]. The physical explanation and mathematical description of this motion were first given by Einstein: it is due to the fluctuations in the force exerted on the Brownian particle by its collisions with the fluid atoms and is described by a diffusion equation for the displacement of the Brownian particle. The ultimate mathematical idealization of this phenomenon is the Ornstein-Uhlenbeck process for the position and velocity of the Brownian particle $(\underline{X}_t, \underline{V}_t)$, described by the stochastic differential equations

$$d\underline{X}_t = \underline{V}_t dt, \tag{0.1}$$

$$d\underline{V}_t = -a\underline{V}_t dt + \sqrt{D}\, d\underline{W}_t, \quad a \geqq 0, \quad D \geqq 0, \quad \underline{W}_t = \text{Wiener process}. \tag{0.2}$$

The position process $\underline{X}_t$ converges in an appropriate limit (e.g. $a \to \infty$, $a^2/D = \text{const}$) to a Wiener process.

A little thought shows that in order to rigorously derive the Ornstein-Uhlenbeck process for the Brownian particle from the mechanical motion of the

* On leave of the Institut für Theoretische Physik I der Universität Münster. Supported by a Nato fellowship

** Supported by NSF Grant, No. PHY 78-03816

*** Supported by NSF Grant, Phy 78-15920

0010-3616/81/0078/0507/$04.80