Commun. Math. Phys. 78, 499–506 (1981)

Communications in
Mathematical
Physics

Relativistic String Model in a Space-Time of a Constant Curvature

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Abstract. The relativistic string model is investigated in a space-time of a constant curvature (de Sitter universe). The fundamental differential quadratic forms of the world surface of the string are considered as the dynamical variables. The coefficients of these forms obey two nonlinear equations

$$\varphi_{,11}-\varphi_{,22}=e^{\varphi}\cos\theta+Ke^{-\varphi},\ \theta_{,11}-\theta_{,22}=e^{\varphi}\sin\theta.$$

The Lax representation for this system is obtained.

1. Introduction

Relativistic string model has rather a long history (see e.g. [1–3]). In the elementary particle physics the relativistic string was introduced as the dynamical basis of the dual resonance models. In recent years the ideas of the string model are used by the investigation of the mechanism of the quark confinement in hadrons [4, 5] and by the representation of the Yang-Mills field theory in terms of the functionals defined on contours [6–9].

In all papers devoted to the relativistic string model the flat space-time was considered. We shall investigate this model in a space-time of a constant curvature (de Sitter universe). If we take the viewpoint that the gravitation may play an important role in the world of the elementary particles (see e.g. [10]), then the aim of this paper will not be percieved as the abstract pure mathematical problem.

We shall use the differential geometry methods when the world surface of the string is described by the differential quadratic forms rather than by the string coordinates [11–15]. In this approach the string dynamics is defined by a system of two non-linear equations. The differential geometry technique enables us to construct the Lax representation for this system (more precisely, the so-called "Zero-curvature equation" [16]).

2. Minimal World Surfaces in de Sitter Space-Time

The Nambu-Goto action of the relativistic string [1–3] can be easily generalized to the curved space-time [17]

$$S=-\kappa\iint d^2u\sqrt{-\det\left\|\frac{\partial x^{\mu}}{\partial u^{i}}\frac{\partial x^{\nu}}{\partial u^{j}}g_{\mu\nu}(x)\right\|}=-\kappa\iint d^2u\sqrt{-\det\|a_{ij}\|}\,, \tag{1}$$

0010-3616/81/0078/0499/$01.60