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Statistical Properties of Lorentz Gas with Periodic Configuration of Scatterers

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Abstract. In our previous paper Markov partitions for some classes of dispersed billiards were constructed. Using these partitions we estimate the decay of velocity auto-correlation function and prove the central limit theorem of probability theory and Donsker's Invariance Principle for Lorentz Gas with periodic configuration of scatterers.

1. Introduction

We consider in this paper the dynamical system which corresponds to the motion of a single particle between fixed scatterers on the plane R^2. Outside all scatterers the particle moves with the constant velocity and at the moments of reflections it changes its velocity according to the usual law of elastic collisions.

We assume that scatterers are disks of arbitrary diameters and the configuration of scatterers is invariant under a discrete subgroup Γ with a compact fundamental domain of the group of all translations of the plane. The fundamental domain of Γ can be chosen as a semi-open set the closure of which is a rectangular. We shall denote it by

$$\Pi=\{q=(q_1,q_2)|0\leqq q_1<B_1,\ 0\leqq q_2<B_2\}.$$

Another assumption concerns the existence of a constant A such that the length of any straight segment which avoids all scatterers cannot be more than A. Sometimes the last property is called as the property to have a finite horizon (see [1]).

The phase space $\mathscr{M}$ of our dynamical system consists of points $x=(q,v)$, where $q=(q^1,q^2)$ are coordinates, $v=(v^1,v^2)$ is velocity of the particle. Without any loss of generality we can restrict ourselves by the case $\|v\|=\sqrt{(v^1)^2+(v^2)^2}=1$. The flow corresponding to our dynamical system will be denoted by $\{S^t\}$. In Theorem 1 we consider a natural special representation of the flow $\{S^t\}$. Namely let $\mathscr{M}_1$ be the space of points $x=(q,v)$ such that q belongs to the boundary of one of the scatterers and v is directed inside the scatterers. We denote by T_0 the transformation of $\mathscr{M}_1$ into itself which arises when the point $x\in\mathscr{M}_1$ moves along its trajectory till the

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