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Symmetry and Bifurcations of Momentum Mappings*

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Abstract. The zero set of a momentum mapping is shown to have a singularity at each point with symmetry. The zero set is diffeomorphic to the product of a manifold and the zero set of a homogeneous quadratic function. The proof uses the Kuranishi theory of deformations. Among the applications, it is shown that the set of all solutions of the Yang-Mills equations on a Lorentz manifold has a singularity at any solution with symmetry, in the sense of a pure gauge symmetry. Similarly, the set of solutions of Einstein's equations has a singularity at any solution that has spacelike Killing fields, provided the spacetime has a compact Cauchy surface.

1. Introduction

A momentum mapping is the conserved quantity associated with a symmetry group acting on phase space. The purpose of this paper is to study the level sets of a momentum mapping and, especially, the zero set. The main results of the paper show that *these level sets have cone-type singularities at any point (in phase space) which itself has some symmetries.*

Level sets of momentum mappings are important in several contexts.

a) The Topology of Hamiltonian Systems with Symmetry

The momentum mapping is conserved by a given Hamiltonian system with symmetry, so knowledge of the level sets and their bifurcations can help in understanding the qualitative features of its flow, as has been emphasized by Smale (1970). In this context, one can reduce a Hamiltonian system with symmetry, looking at the orbit space of a level set of the momentum mapping. This general

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