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The Decay of the Gravitational Field

B. G. Schmidt

Max-Planck-Institut für Physik und Astrophysik, Institut für Astrophysik, D-8046 Garching, Federal Republic of Germany

Abstract. The asymptotic behaviour of Einstein-Rosen waves, a class of nonstationary solutions of Einstein's vacuum equations, is investigated. It is established that solutions of this type exist which admit part of $\mathscr{I}^+$ and a regular I^+ in the sense of Penrose.

1. Introduction

In classical field theories it is important to know how the field decays. Does the free field at a fixed space point always ultimately tend to zero if time goes to infinity? Are there soliton-like solutions? Answeres to questions of this nature contain extremely valuable information about the field.

For Einstein's field equations it is very hard to give a precise meaning to such questions, and even more to find an answer. The equations are nonlinear, but in contrast to nonlinear field theories on Minkowski-space, the field itself determines the spacetime geometry! Even to find "where infinity is" is a nontrivial matter!

The decay behaviour of linear equations is much simpler. Therefore it is natural, in the case of nonlinear field theories on Minkowski space, to ask the question whether solutions exist for which the field becomes small for $t \to \infty$ and tends to a solution of the linearised equations, the nonlinear effects becoming irrelevant. This leads to a scattering theory in the sense of Lax and Phillips [1].

To follow a similar procedure for Einstein's equations, one has first to formulate in which sense a spacetime becomes asymptotically Minkowskian, and then to establish that there exist solutions having this property. The first problem was posed by Bondi et al. [2] and Penrose [3], leading to the concept of spacetimes admitting null infinity $\mathscr{I}$. A spacetime with this property becomes Minkowskian along null geodesics, and the curvature tensor approaches a solution of the linear spin-two field equations.

The compatibility of these conditions with the field equations remained however rather unclear.

In this paper I examine the asymptotic behaviour of a well-known class of local solutions of Einstein's vacuum field equations, the Einstein-Rosen waves [4, 5]. It

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