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The Local Structure of the Spectrum of the One-Dimensional Schrödinger Operator

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Abstract. Let $H_V = -\frac{d^2}{dt^2} + q(t,\omega)$ be an one-dimensional random Schrödinger operator in $\mathscr{L}^2(-V, V)$ with the classical boundary conditions. The random potential $q(t,\omega)$ has a form $q(t,\omega) = F(x_t)$, where x_t is a Brownian motion on the compact Riemannian manifold K and $F: K \to R^1$ is a smooth Morse function, $\min_K F = 0$. Let $N_V(\Delta) = \sum_{E_i(V)\in\Delta} 1$, where $\Delta \in (0,\infty)$, $E_i(V)$ are the eigenvalues of H_V. The main result (Theorem 1) of this paper is the following. If $V \to \infty$, $E_0 > 0$, $k \in Z_+$ and $a > 0$ (a is a fixed constant) then

$$P\left\{N_V\left(E_0 - \frac{a}{2V}, E_0 + \frac{a}{2V}\right) = k\right\} \xrightarrow[V\to\infty]{} e^{-an(E_0)}(an(E_0))^k|k!,$$

where $n(E_0)$ is a limit state density of H_V, $V \to \infty$. This theorem mean that there is no repulsion between energy levels of the operator H_V, $V \to \infty$.

The second result (Theorem 2) describes the phenomen of the repulsion of the corresponding wave functions.

1.

In a series of latest works in physics (see [1]) the phenomenon of the repulsion of the energy levels in the spectrum of complicated (random) quantum systems was discussed. The formal definitions are the following.

Let $\mathbf{H}_V$ be the family of the Hamiltonians describing the behaviour of the system in the volume V and let $E_1^{(V)} < E_2^{(V)} \leqq \ldots$ be the corresponding energy levels. In various interesting cases these levels are thickening in the limit and moreover for every $\alpha > 0$ $E_{\alpha|V|}^{(V)} \to E_\alpha$ as $|V| \to \infty$.

We shall consider two neighbour levels $E_n^{(V)}$ and $E_{n+1}^{(V)}$, where $n \sim \alpha|V|$. It is natural to suppose that the normalized "spectral split" $\Delta_n = (E_{n+1} - E_n)/M(E_{n+1} - E_n)$ has a limit distribution as $|V| \to \infty$, i.e. there exists

$$\lim_{|V|\to\infty} \mathbf{P}\{\Delta_n < x\} = G(x).$$

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