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Some Nonabelian Toy Models in the Large N Limit

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Abstract. Schwinger–Dyson equations are used to study the large N limit of $U(N)$ gauge theory on several small lattices. Explicit solutions are found which are beyond the reach of existing steepest descent technique. They show a phase transition in a three placquette model at coupling $g^2N = 3$, resembling the known transition in the one placquette model, and lending support to expectations of a similar transition in the four dimensional lattice theory.

1. Introduction

The $U(\infty)$ lattice gauge theory differs qualitatively from the $U(N)$ or $SU(N)$ theories in having an infinite number of internal degrees of freedom per unit volume of space-time. Gross and Witten [1], using the steepest descent technique of [2], solved exactly the large N limit of $U(N)$ gauge theory on a single placquette and found peculiarities attributable to just this difference. The free energy and correlation functions depend analytically on the coupling constant except at a single critical value, which marks a continuous transition between weak and strong coupling phases. The average eigenvalue distribution of the placquette variable (an $N \times N$ unitary matrix) covers the entire unit circle in the strong coupling regime, but for small coupling constant is excluded from a neighborhood of -1.

This paper presents some new exact results for $U(\infty)$ gauge theories on small lattices. The main result is for a three placquette model which consists essentially of two unitary matrices governed by the action $S(U_1, U_2) = -\beta N \operatorname{tr}(U_1 + U_2 + U_1U_2^* + \text{adjoints})$. The steepest descent method (in the form used in [1] and [2]) fails here because the number of true degrees of freedom goes as N^2 rather than N. We look instead to the Schwinger–Dyson equations recently derived for lattice gauge theories [3, 4]. Because of symmetries special to the three placquette lattice, the $N = \infty$ Schwinger–Dyson equations close on a manageable subset of the correlation functions. Extending a technique suggested by Foerster [5],

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