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Cohomology and Massless Fields*

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Introduction

The geometry of twistors was first introduced in Penrose [28]. Since that time it has played a significant role in solutions of various problems in mathemetical physics of both a linear and nonlinear nature (cf. Penrose [29], Penrose [35], Ward [48], and Atiyah–Hitchin–Drinfeld–Manin [2], see Wells [52] for a recent survey of the topic with a more extensive bibliography). The major role it has played has been in setting up a general correspondence which translates certain important physical field equations in space-time into holomorphic structures on a related complex manifold known as *twistor space*. The purpose of this paper is to give a rigorous discussion of this correspondence for the case of the linear massless free-field equations, including Maxwell's source-free equations, the wave equation, the Dirac–Weyl neutrino equations, and the linearized (weak-field limit of) Einstein's vacuum equations. These equations may also be analyzed from this point of view on a background provided by the nonlinear Yang–Mills or Einstein equations in the (anti-) self-dual case. The correspondence is effected by an integral-geometric transform, which transforms complex-analytic data on twistor space to solutions of the linear massless field equations, and is, in fact, a generalization of the classical Radon transform, which is discussed further below.

The motivation for finding such a correspondence in general is that it forms an essential part of the "twistor programme" according to which one attempts to eliminate the equations of physics by deriving them from the rigidity of complex geometry and holomorphic functions (see, e.g. Penrose [38]). It is, in fact, rather remarkable the extent to which it is possible to achieve this. Success apparently comes about because in twistor-space descriptions the information is "stored" nonlocally. The (local) value of a field at a point in space-time depends upon the way that the holomorphic structure in the twistor-space is fitted together in the large. So sheaf cohomology and function theory of several complex variables turn out to be the appropriate tools in the twistor framework. It is hoped that, as part of the general twistor programme, some deeper insights may eventually be gained as to the interrelation between quantum mechanics or quantum field

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