Commun. Math. Phys. 78, 303 (1980)

Communications in Mathematical Physics

Erratum

Equilibrium States for a Plane Incompressible Perfect Fluid

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Commun. Math. Phys. **72**, 55–76 (1980)

On p. 57, line 9, "Let Ω denote an open ..." should read "Let Ω denote a bounded open ..."

On p. 58, last line, "if **u** is a classical ..." should read "if **u** is a sufficiently smooth classical ..."

On pp. 67–68 the proof of Theorem 4.2 does not work as it is, since $\tilde{\mathbf{m}}$ is not in general in $\bar{\mathbb{Z}}^2$. The proof should be modified as follows:

For any positive integers $N', N'', N' > N''$, we have

$$\|B_{\mathbf{k}}^{(N')} - B_{\mathbf{k}}^{(N'')}\|^2_{L^2(d\mu_\psi)} = 2(\psi''(0))^2 \sum_{\mathbf{m} \in I_{N'}(\mathbf{k}) \setminus I_{N''}(\mathbf{k})} (\tilde{\Gamma}_{\mathbf{m},\mathbf{k}})^2 + \psi^{\mathrm{IV}}(0) \Big(\sum_{\mathbf{m} \in I_{N'}(\mathbf{k}) \setminus I_{N''}(\mathbf{k})} \tilde{\Gamma}_{\mathbf{m},\mathbf{k}} \Big)^2 \tag{1}$$

with $\tilde{\Gamma}_{\mathbf{m},\mathbf{k}} = \Gamma_{\mathbf{m},\mathbf{k}}/(m|\mathbf{k}-\mathbf{m}|)$ and $I_N(\mathbf{k}) = \{\mathbf{m} \in \bar{\mathbb{Z}}^2 | \mathbf{m}, \mathbf{k}-\mathbf{m} \in I_N\}$. Setting $F_N(\mathbf{k}) = \sum_{\mathbf{m} \in I_N(\mathbf{k})} \tilde{\Gamma}_{\mathbf{m},\mathbf{k}}$ and $\mathscr{I}_N(\mathbf{k}) = \{\mathbf{m} \in \bar{\mathbb{Z}}^2 | m \leqq N, |\mathbf{k}-\mathbf{m}| > N\}$ we get $F_N(\mathbf{k}) = \sum_{m \leqq N} \tilde{\Gamma}_{\mathbf{m},\mathbf{k}} - \sum_{\mathbf{m} \in \mathscr{I}_N(\mathbf{k})} \tilde{\Gamma}_{\mathbf{m},\mathbf{k}}$. The last sum tends to 0 and the limit $F(\mathbf{k}) = \lim_{N \to \infty} F_N(\mathbf{k})$ exists because the series $\sum_{\mathbf{m} \in \bar{\mathbb{Z}}^2} |\tilde{\Gamma}_{\mathbf{m},\mathbf{k}} + \tilde{\Gamma}_{\mathbf{m}^\perp,\mathbf{k}}|$ converges. Since $\sum_{\mathbf{m} \in \bar{\mathbb{Z}}^2} (\tilde{\Gamma}_{\mathbf{m},\mathbf{k}})^2 < \infty$, $B_{\mathbf{k}}^{(N)}$ is Cauchy in $L^2(d\mu_\psi)$. Using (1), and observing that $(B_{\mathbf{k}}^{(N')}, B_{\mathbf{k}'}^{(N'')})_{L^2(d\mu_\psi)} = 0$ for $\mathbf{k} \neq \mathbf{k}'$ and any N', N'', one gets that $\langle B^{(N)}, \hat{\theta} \rangle$ is $L^2(d\mu_\psi)$-convergent $\forall \hat{\theta} \in \tilde{s}$. Convergence almost everywhere is shown only in the gaussian case. To show that $F(\mathbf{k}) = 0 \, \forall \mathbf{k} \in \bar{\mathbb{Z}}^2$, which completes the proof, observe that

$$F_N(\mathbf{k}) = \frac{1}{k} \sum_{\mathbf{m} \in I_N(\mathbf{k})} \frac{\mathbf{m}^\perp \cdot \mathbf{k}}{m^2} = \frac{1}{k} \Big(\sum_{m \leqq N} - \sum_{\mathbf{m} \in \mathscr{I}_N(\mathbf{k})} \Big) \frac{\mathbf{m}^\perp \cdot \mathbf{k}}{m^2} = -\frac{1}{k} \sum_{\mathbf{m} \in \mathscr{I}_N(k)} \frac{\mathbf{m}^\perp \cdot \mathbf{k}}{m^2},$$

the first sum vanishing by antisymmetry. By elementary geometric considerations one gets $F(n\mathbf{k}) = nF(\mathbf{k})$, $\mathbf{k} \in \bar{\mathbb{Z}}^2$, $n \geqq 0$ integer. The proof is concluded by the estimate $|F_N(n\mathbf{k})| < c$, for N large enough, c a constant independent of n (which is obtained, e.g., by estimating the sum $F_N(n\mathbf{k}) + \int_{\substack{|\mathbf{x}| \leqq N \\ |\mathbf{x} - n\mathbf{k}| > N}} \frac{\mathbf{x}^\perp \cdot \mathbf{k}}{k|\mathbf{x}|^2} dx$, the integral being 0 by antisymmetry). The proof of Theorem 6.1 needs a small obvious modification.

0010-3616/80/0078/0303/$01.00