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Lattice Systems with a Continuous Symmetry

I. Perturbation Theory for Unbounded Spins

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Abstract. We investigate a continuous Ising system on a lattice, equivalently an anharmonic crystal, with interactions:

$$\sum_{\langle x,y\rangle} (\phi_x-\phi_y)^2+\lambda(\phi_x-\phi_y)^4, \qquad \phi_x\in\mathbb{R}, \qquad x\in\mathbb{Z}^d.$$

We prove that the perturbation expansion for the free energy and for the correlation functions is asymptotic about $\lambda=0$, despite the fact that the reference system ($\lambda=0$) does not cluster exponentially. The results can be extended to more general systems of this type, e.g. an even polynomial semi-bounded from below instead of a quartic interaction. By a suitable scaling, λ corresponds to the temperature.

I. Introduction

In recent years there have been several works giving a mathematical justification to the high and low temperature (H.T., L.T.) perturbation theory frequently used by physicists in statistical mechanics and quantum field theory, see for example [4, 6–9, 11, 13].

In all these cases the perturbation is made around an unperturbed system which is explicitly known, e.g. a Gaussian field ($\lambda P(\Phi)_2$ theories) or a product of uncoupled systems (statistical mechanics in H.T. regime), and which is massive in the sense that its correlation functions are exponentially clustering. No similar justification has been given, however, for the case where the reference system is not exponentially clustering. This occurs for example in the anharmonic crystal and the n component Heisenberg (classical or quantum) spin system. In both cases the lack of exponential clustering appears related to the invariance of the Hamiltonian

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