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Markov Partitions for Dispersed Billiards*

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Abstract. Markov Partitions for some classes of billiards in two-dimensional domains on $\mathbb{R}^2$ or two-dimensional torus are constructed. Using these partitions we represent the microcanonical distribution of the corresponding dynamical system in the form of a limit Gibbs state and investigate the character of its approximations by finite Markov chains.

1. Dispersed Billiards and Formulation of Main Results

Let Q be a two-dimensional open bounded connected domain on $\mathbb{R}^2$ or the two-dimensional torus with Euclidean metric. We suppose that the boundary ∂Q consists of a finite number of C^3-smooth non-selfintersecting curves Γ_i, $i=1,2,\dots,p$, which may be either closed or have common end-points.

Billiard in Q is the dynamical system which corresponds to the motion of a material point inside Q by inertia with elastic reflections at the boundary.

We consider the framing of each Γ_i by unit normal vectors $n(q)$, $q\in\Gamma_i$, directed inside Q. As a result the curvature of each Γ_i takes a definite sign. Dispersed billiards are billiards for which all Γ_i have a strictly positive curvature (see [1]).

Let M be the unit tangent bundle over Q, π is the natural projection of M onto Q. Preimage $\pi^{-1}(q)=S^1(q)$, $q\in Q$ consists of unit vectors which are tangent to Q at $q\in Q$. M is the three-dimensional open manifold with the boundary $\partial M=\bigcup_{i=1}^{p}\pi^{-1}(\Gamma_i)=\bigcup_{i=1}^{p}\partial M_i$. On every ∂M_i one can introduce natural coordinates (r,φ) where r is the parameter of length on every Γ_i and φ is the angle between x and $n(q)$, $q=\pi(x)$. Let

$$M_1=\{x\in\partial M:(x,n(q))\geqq 0,\ q=\pi(x)\},\qquad M_1^{(i)}=M_1\cap\partial M_i$$

$$S_0=\{x\in\partial M:(x,n(q))=0,\ q=\pi(x)\},$$

$$M_2=\bigcup_{i\neq j}\pi^{-1}(\Gamma_i\cap\Gamma_j),\qquad M_s=S_0\cup M_2.$$

* Dedicated to the memory of Rufus Bowen

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