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Mass Spectrum of the Two Dimensional $\lambda\phi^4 - \frac{1}{4}\phi^2 - \mu\phi$ Quantum Field Model

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Abstract. It is shown that r-particle irreducible kernels in the two-dimensional $\lambda\phi^4 - \frac{1}{4}\phi^2 - \mu\phi$ quantum field theory have $(r+1)$-particle decay for $|\mu| \leqq \lambda^2 \ll 1$. As a consequence there is an upper mass gap and, in the subspace of two-particle states, a bound state. The proof extends Spencer's expansion [20] to handle fluctuations between the two wells of the classical potential. A new method for resumming the low temperature cluster expansion is introduced.

Introduction

Much progress has been made recently in describing in detail pure phases of quantum field models in low temperature regions of coupling. Glimm et al. [16] developed a convergent expansion for the Schwinger functions of the $\lambda\phi^4 - \frac{1}{4}\phi^2 - \mu\phi$ model in two dimensions (with $|\mu| \leqq \lambda^2 \ll 1$), establishing also the mass gap of the theory. Subsequently their expansion technique has been applied to some ϕ_2^6 models with three minima [22, 23], to the two-dimensional pseudoscalar Yukawa model in the two-phase region [1], and to the Coulomb gas in the sine-Gordon representation [2, 3]. Investigators have concentrated on proving the cluster property of correlations and the mass gap, leaving the higher spectrum unexplored.

A wealth of information is known about the spectrum of single phase $\lambda P(\phi)_2$ theories with λ small. The n-particle cluster expansion [14] was used to establish the existence of isolated one-particle states and to show that for $\lambda < \lambda(n, \varepsilon)$, n field operators are sufficient to generate all states of energy less than $(n+1)m(1-\varepsilon)$, where m is the single particle mass. Spencer [20] introduced an expansion for r-particle irreducible kernels, proving $(r+1)$-particle decay. For even theories this information was used to analyze the mass spectrum below $3m - \varepsilon$ [8, 9, 21], with results including asymptotic expansions for bound state masses and scattering amplitudes, and asymptotic completeness in this energy region. Burnap [5] showed (without resorting to the n-particle cluster expansion) that in general

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