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Low Temperature Expansion for Continuous-Spin Ising Models*

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Abstract. We consider general ferromagnetic spin systems with finite range interactions and an even single-spin distribution of compact support on $\mathbb{R}$. It is shown under mild assumptions on the single-spin distribution that a low temperature expansion, in powers of T, for the free energy and the correlation functions is asymptotic. We also prove exponential clustering in the pure phases and analyticity of the free energy and of the correlation functions in the reciprocal temperature β for $\mathrm{Re}\,\beta$ large.

I. Introduction

In this paper we develop a low temperature expansion for bounded spins on $\mathbb{R}$ distributed with a continuous measure e.g. a uniform distribution on $[-1, +1]$ and nearest neighbour interactions Js_is_j, $J>0$. As a consequence, we obtain analyticity and exponential clustering of the correlation functions at low temperatures. We also obtain asymptotic series in powers of T for various quantities.

The bounded continuous spin system is somewhat intermediate between the Ising model and the Field Theory case studied respectively in [1, 2]. In the Ising model the low temperature expansion is in terms of contours: to each configuration one associates a family of connected contours i.e. lines separating + and − regions. These are pairwise disconnected and summing over configurations amounts to considering all possible arrangements of connected contours. In Field Theory one deals with continuous variables and it is impossible to associate uniquely configurations and contours. Instead, one first uses a contour expansion in order to isolate regions of pure phases (separated by contours) where the field sits in one of the "potential wells". It is then necessary to supplement this contour expansion by an expansion in the pure phases (away from the contours) which is a

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