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Structure of Gibbs States of one Dimensional Coulomb Systems

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Abstract. We present a method of computing the Coulomb forces on particles in an infinite configuration of charges in one dimension. The resolution of the apparent nonuniqueness in this problem leads to a structural proof of the translation symmetry breaking in jellium, at all temperatures, and to a related phenomenon of phase nonuniqueness in the two component system. The appropriate generalizations of the DLR and KMS conditions for these states are discussed.

1. Introduction

Coulomb systems are of interest even in one dimension since:

1) Coulomb forces play such a fundamental role in Nature.

2) The systems offer tractable examples of situations in which the infinite range of the interaction poses difficulties already in the formulation of the laws of motion and of the conditions which are generally used to describe equilibrium states (KMS, DLR). The resolution of these difficulties requires the introduction of new methods which take into account the collective effect of the bulk system.

3) Some of the systems exhibit symmetry breaking and others may exist in various phases. The two component systems will be shown to admit states with a non vanishing electric field. Our method offers a unified treatment of this phenomenon and of the translation symmetry breaking in jellium, which in [1, 2] was proven to form a Wigner-lattice.

The one dimensional Coulomb interaction energy of particles with charges σ_i located at $q_i \in \mathbb{R}$, is:

$$H(q,\sigma) = -1/2 \sum_{i,j} \sigma_i \sigma_j |q_i - q_j| , \tag{1.1}$$

with the corresponding electric field at $x \in \mathbb{R}$:

$$E(x;q,\sigma) = \sum_j \sigma_j \operatorname{sgn}(x - q_j) . \tag{1.2}$$

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