Commun. Math. Phys. 78, 83–98 (1980)

Communications in Mathematical Physics

Gravitating Fermions in an Infinite Configuration Space*

A. Pflug

Institut für Theoretische Physik, Universität Wien, Boltzmanngasse 5, A-1090 Wien, Austria

Abstract. For a system of (infinitely many) nonrelativistic gravitating fermions described rigorously by Thomas-Fermi theory, a nontrivial limit of infinite configuration volume $|\Lambda|$ is shown to exist for the microcanonical free energy, and for the entropy divided by $\log|\Lambda|$. It can be calculated explicitly using the scaling behaviour of the (ground state). Thomas-Fermi equation and shows a phase transition at zero energy. For all (possible) negative energies, the heat capacity of the infinitely extended system is negative and a nonzero fraction of the particles is in the condensed phase.

0. Introduction

For a system of N nonrelativistic gravitating fermions, a nontrivial limit $N\to\infty$ exists for the entropy (see [4]), the free energy [5] and the thermodynamical pressure [8], if those functions, together with their arguments, are appropriately scaled with N (see [4]). For nonzero temperatures, the system has to be enclosed in a "box" = bounded open region $\Lambda\subseteqq\mathbb{R}^3$ whose linear dimensions have to shrink proportional to $N^{-1/3}$ in order to give a nontrivial limit for the collapsing system. To work with an N-independent confining region (which is conceptually simpler) we choose an N-dependent length scale and replace the original Hamiltonian [with units $h=(4\pi)^{1/3}$, $m=1/2$, gravitational constant $\kappa=1/\pi$]

$$H_{N,\Lambda}=2^{-2/3}\pi^{-4/3}\sum_{i=1}^{N}(-\Delta_i)-\frac{1}{4\pi}\sum_{i<j}|x_i-x_j|^{-1}$$

by

$$H_{N,\Lambda}^{\sim}=N^{-4/3}U^{\dagger}H_{N,\Lambda_N}U=N^{-2/3}2^{-2/3}\pi^{-4/3}\sum_{i=1}^{N}(-\Delta_i)-\frac{1}{4\pi N}\sum_{i<j}|x_i-x_j|^{-1}$$

$$(U^{\dagger}x\restriction\Lambda_N U=N^{-1/3}x\restriction\Lambda),$$

* Work supported in part by Fonds zur Förderung der wissenschaftlichen Forschung in Österreich, Project No. 3569

0010-3616/80/0078/0083/$03.20