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Proof of a Multipole Conjecture due to Geroch

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Abstract. A result, first conjectured by Geroch, is proved to the extent, that the multipole moments of a static space-time characterize this space-time uniquely. As an offshoot of the proof one obtains an essentially coordinate-free algorithm for explicitly writing down a geometry in terms of it's moments in a purely algebraic manner. This algorithm seems suited for symbolic manipulation on a computer.

1. Introduction

In the literature on General Relativity there have been several approaches to multipole moments in the static or more general context, e.g. the one of Clarke and Sciama [1], based on a certain eigenvalue problem. Another one, due to Geroch [2, 3], which uses a conformal compactification of 3-space, has found special attention because of the elegant geometric way in which the origin-dependence of multipole moments can be expressed. Our work is based on Geroch's approach.

Multipole moments in General Relativity provide, or should provide, an invariant way of interpreting exact or approximate solutions to Einstein's equations. Of course, this is true only if such solutions are uniquely determined by these moments up to isometries. The latter statement, which is the content of Geroch's "Conjecture 1" [3], is proved in the present paper. Partial results on this question have before been obtained by Xanthopoulos [4].

The strategy of our proof parallels the corresponding Newtonian proof. There one proceeds in two steps: In Step 1 one shows that the so-called Kelvin transform, which is the flat-space analogue of Geroch's compactification trick leaves the field equation, namely Laplace's equation, invariant. Therefore the potential, if it goes to zero for large distances from the source, is analytic near the origin Λ of the compactified space. In Sect. 3 we prove an analogous result within Einstein's theory. In the finite region the analyticity of the field variables is, in fact, well known [5]. The situation is considerably trickier at infinity, due to the fact that Einstein's equations, far from being conformally invariant, become formally singular at the point Λ. (One faces a similar problem in the characteristic initial value problem at null infinity [6].) The essential idea to overcome this

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