Commun. Math. Phys. 77, 269–288 (1980)

Communications in
Mathematical
Physics

Poisson Processes on Groups and Feynman Path Integrals

Ph. Combe[1], R. Høegh-Krohn[4], R. Rodriguez[1], M. Sirugue[2], and M. Sirugue-Collin[3]

[1] Faculté des Sciences de Luminy and Centre de Physique Théorique
[2] Centre de Physique Théorique
[3] Université de Provence and Centre de Physique Théorique, CNRS, Marseille, France, and
[4] Matematisk Institutt, Universitet i Oslo, Oslo

Abstract. We give an expression for the perturbed evolution of a free evolution by gentle, possibly velocity dependent, potential, in terms of the expectation with respect to a Poisson process on a group. Various applications are given in particular to usual quantum mechanics but also to Fermi and spin systems.

Introduction

Since the proposal of Feynman [1] of a quantization procedure by means of path integrals, a great deal of effort has been devoted to make precise the definition of such an integral (see e.g., [2, 3] for references). An important step has been to realize that in some cases this path integral can be interpreted as the expectation with respect to a stochastic process [4]. One of the most natural stochastic processes which has been considered first is the Gaussian process. Another one is the Poisson process [5, 6]. In this paper, we shall consider the connection between Poisson processes on groups and the usual Weyl quantization procedure. More specifically, let H_0 be the free hamiltonian of a single particle and V a multiplicative potential which is the Fourier transform of a bounded measure ν. In the "p" representation H_0 is just a multiplication operator by $h_0(p)=p^2$, then for any square integrable, continuous function ψ one has

$$\{e^{-iT(H_0+V)}\psi\}(p)=E^T_{p(T)=0}\left\{e^{-i\int_0^T h_0(p-p(\tau))d\tau}\psi(p-p(0))\right\},$$

where $E^T_{p(T)=0}$ denotes the expectation value with respect to a Poisson process naturally associated to ν, Proposition (3.7).

In fact this result is a special case of a more general result. Let G be a topological group which acts on a topological space $\mathfrak{X}$, and let μ be a quasi-invariant measure on $\mathfrak{X}$ with respect to the action of G. Under these assumptions the action of G on $\mathfrak{X}$ induces $*$-automorphisms of $L_\infty(\mathfrak{X},\mu)$ which are unitarily implemented by a unitary projective representation U of G. Let $\mathfrak{M}$ be the cross product of $L_\infty(\mathfrak{X},\mu)$ by the action of G. Then it is possible to write a formula of the

0010-3616/80/0077/0269/$04.00