Commun. Math. Phys. 77, 251–267 (1980)

Communications in
Mathematical
Physics

Macrocausality, Unitary and Physical-Region Structure of the Multiparticle S Matrix

D. Iagolnitzer

Service de Physique Théorique, Cen-Saclay, F-91190 Gif-sur Yvette, France

Abstract. The physical-region analytic structure of the S-matrix is established on the basis of refined macrocausality, unitary and a weak supplementary analyticity assumption for a $3 \to 3$ process below the four-particle threshold, and away from points where two initial, or two final, four-momenta are colinear: the connected S matrix is decomposed as a sum of contributions that generalize Feynman graphs and are associated with the various $+\alpha$-Landau surfaces encountered (= surfaces of graphs with one internal line and triangle graphs). The property of "separation of singularities" in unitarity equations, which was used as a crucial ad hoc assumption in traditional derivations of related results in S-matrix theory, is obtained as a byproduct in the region considered.

1. Introduction

This work is a continuation of recent works, carried out in the mathematical framework of essential support theory, or of hyperfunction theory, and aiming at the derivation in S-matrix theory, from basic principles, of the physical-region analytic structure of multiparticle collision amplitudes between sets of massive particles with short-range interactions.

Such a derivation was first attempted in the sixties [1] on the basis of unitarity and "maximal analyticity" [2], interpreted, when applied to the physical region, as providing analyticity of scattering functions (= connected momentum-space collision amplitudes, after factorization of their energy-momentum conservation δ-functions) outside Landau surfaces of connected graphs, and the plus $i\varepsilon$ rule at $+\alpha$-Landau points. The aim was then to establish the detailed structure at $+\alpha$-Landau points, in the form of à la Cutkosky discontinuity formulae (with S-matrices associated to each vertex), as also analyticity outside the $+\alpha$-parts of the Landau surfaces. To that purpose, the general idea is to analyze the possible singularities of the various "bubble diagram functions" (= integrals, over internal on-mass-shell four-momenta, of products of connected kernels of S or $S^{-1} = S^{\dagger}$) that occur in unitarity equations, and to use the fact that their singularities must cancel among themselves.

0010-3616/80/0077/0251/$03.40