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Algebras of Local Observables on a Manifold*

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Abstract. We propose a generalization of the Haag–Kastler axioms for local observables to Lorentzian manifolds. The framework is intended to resolve ambiguities in the construction of quantum field theories on manifolds. As an example we study linear scalar fields for globally hyperbolic manifolds.

1. Axioms

Quantum field theories are usually defined on Minkowski space-time, but it seems desirable to generalize to arbitrary Lorentzian manifolds. This is so not only to accommodate physical systems that require a manifold model for space-time, but also as a means of gaining perspective on the general structure of quantum field theories. General references for quantization on manifolds are De Witt [3] and Isham [12].

The problem can be posed as finding field operators which satisfy given field equations. However, on a general manifold there is no natural choice for the Hilbert space on which the operators act, and, in particular, there is no vacuum state to be used as a reference point. This suggests formulating the problem in terms of the algebraic structure of the field operators, and leaving the specification of states as a secondary step.

One algebraic approach has been developed by Isham [12], Kay [13], and Hajicek [10]. They associate with each Cauchy surface S the C^* algebra $\mathscr{A}_S$ generated by the canonical commutation relations (CCR) over functions on S. The field equations then determine isomorphisms $\mathscr{A}_S \to \mathscr{A}_{\hat{S}}$ which give the dynamics. This type of approach seems quite satisfactory for linear problems, but one can anticipate troubles in extending it to nonlinear problems: things are probably too singular to allow a definition of the algebras $\mathscr{A}_S$.

In this paper we propose another algebraic approach which generalizes the Haag–Kastler algebras of local observables on Minkowski space [8]. There is a single C^* algebra $\mathscr{A}$ together with distinguished subalgebras $\mathscr{A}(\mathcal{O})$ corresponding to local regions of space-time. All reference to fields to supressed.

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