Commun. Math. Phys. 77, 65–86 (1980)

Communications in
Mathematical
Physics

The Transition to Aperiodic Behavior in Turbulent Systems

Mitchell J. Feigenbaum
University of California, Los Alamos Scientific Laboratory (T-DOT MS 452), P.O. Box 1663, Los Alamos, NM 87545, USA

Abstract. Some systems achieve aperiodic temporal behavior through the production of successive half subharmonics. A recursive method is presented here that allows the explicit computation of this aperiodic behavior from the initial subharmonics. The results have a character universal over specific systems, so that all such transitions are characterized by noise of a universal internal similarity.

Introduction

A variety of numerical experiments [1–6] on systems of differential equations have demonstrated that a possible route to chaotic or turbulent behavior is a cascade of successive half harmonics of a basic mode with turbulence commencing after an infinite halving has produced non-periodic behavior. Moreover, a recent experiment [7] on Rayleigh Bénard convection has also exhibited these half harmonics as the behavior determining the onset of turbulence in the fluid. In this paper we draw upon some mathematics intimately connected with period doubling, and determine the time fluctuation spectrum of such a system at the onset of turbulence [8]. This spectrum proves to be of a *universal* construction, so that no specific formulas for the differential system are ever encountered. On the other hand, the theory presented is asymptotic and recursive, so that it requires as input the specific spectrum after several stages of period doubling. We are not concerned with this aspect here, taking this input for granted after which the entirety of the behavior through the transition is computed.

The paper is divided into three main parts. The first *assumes* the theory later to be exposited in order to present the new results with the least dedication required of the reader. The principal results are the formulae (10) and (17) which determine the subharmonic spectrum recursively. The second section reviews the universality theory for one-dimensional maps, and constructs the basic scaling function required in the first part. Finally, the last section consists of an argument establishing the relevance of one-dimensional maps to the original system of differential equations, resting upon very recent work of Collet et al. [9]. The correct formula to which (10) is a rough approximation is also determined.