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Conservation Laws in the Quantum Version of N-Positional Potts Model

Yu. A. Bashilov and S. V. Pokrovsky

L. D. Landau Institute for Theoretical Physics, Academy of Sciences, SU-142432, Chernogolovka, Moscow Region, USSR

Abstract. A quantum analogue of the N-positional Potts model is constructed. The system is shown to possess an infinite set of involutory conservation laws in the phase transition point.

1. Introduction

Most of exactly solvable two-dimensional spin models already known are related to the eight-vertex model, analyzed by Baxter [4]. The unique feature of this model is the existence of a complete set of conserved integrals commuting with each other [6]. The transfer matrix of the original model is at the same time the generating function of these integrals [4, 6]. These quantities can be considered as quantum Hamiltonians describing different one-dimensional systems, one of which coincides with the XYZ-model [5]. It is a natural question whether there exist any other systems possessing a hidden symmetry of the same type.

In the present paper it will be shown that this hidden symmetry is indeed present in a wide class of N-positional Potts models. However, conservation laws exist for a certain relation between the original quantum Hamiltonian constants which apparently corresponds to the phase transition point in statistical mechanics. Thus the only way to investigate the critical behaviour of the model is to compute the correlation functions at the phase transition point. To avoid terminological misunderstanding it should be noted that the generating function of conserved integrals in the quantum version of the Potts model cannot be regarded as a transfer matrix of the classical lattice statistical Potts model [7] and thus no exact relation between these models can be indicated.

We employ the representation of the Heisenberg motion equation using the $L-A$ pair which makes it possible to write down directly the generating functional of conservation laws. We illustrate our method considering the Ising chain in Sect. 2 and the anisotropic XYZ-model in Appendix A. In Sect. 3 we formulate the quantum version of the Potts model in a way convenient for further developments. In Sect. 4 the generating functional of motion integrals is written down, some of

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