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Monodromy- and Spectrum-Preserving Deformations I

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Abstract. A method for solving certain nonlinear ordinary and partial differential equations is developed. The central idea is to study monodromy preserving deformations of linear ordinary differential equations with regular and irregular singular points. The connections with isospectral deformations and with classical and recent work on monodromy preserving deformations are discussed. Specific new results include the reduction of the general initial value problem for the Painlevé equations of the second type and a special case of the third type to a system of linear singular integral equations. Several classes of solutions are discussed, and in particular the general expression for rational solutions for the second Painlevé equation family is shown to be $-\frac{d}{dx}\ln(\Delta_+/\Delta_-)$, where Δ_+ and Δ_- are determinants. We also demonstrate that each of these equations is an exactly integrable Hamiltonian system. The basic ideas presented here are applicable to a broad class of ordinary and partial differential equations; additional results will be presented in a sequence of future papers.

1. Introduction and Outline

This paper is the first in what is planned to be a series of studies on deformations of linear ordinary differential equations with coefficients rational on a Riemann surface. The deformations in question preserve the monodromy at singular points of the linear equation, and this requirement forces the coefficients of the linear equation to satisfy certain nonlinear ordinary or partial differential equations of considerable interest. The theory of monodromy-preserving deformations overlaps the theory of isospectral deformations (i.e., soliton theory), and indeed one of our aims will be to understand the connections between these two types of problems. Applications of the nonlinear equations governing monodromy-preserving deformations have been discovered in nonlinear waves, statistical

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