Commun. Math. Phys. 76, 39–64 (1980)

Communications in
Mathematical
Physics

The Renormalization Group of the Model of A^4-Coupling in the Abstract Approach of Quantum Field Theory

Wolfhart Zimmermann

Max-Planck-Institut für Physik und Astrophysik, München, Federal Republic of Germany

Abstract. For the model of A^4-interaction the postulates of the renormalization group are stated within the abstract approach of quantum field theory. In the massive case these postulates follow if an on-shell formulation of the model is assumed to exist. For the massless model the postulates of the renormalization group imply that the propagator has a pole at momentum zero. Consequently there is no dynamic mass generation and the propagator is normalizable on the mass shell. It is shown that the S-matrix elements scale with canonical dimensions. A general method of rescaling parameter values is developed which takes into account the possibility of propagator zeros and stationary points of the effective coupling.

1. Introduction

Originally the renormalization group was introduced by Petermann and Stueckelberg within the framework of the perturbative treatment of Lagrangian field theory [1]. The work of Gell-Mann and Low, as well as Bogoliubov and Shirkov, has made evident that the concept of the renormalization group goes far beyond perturbation theory [2, 3]. Consequences for the infrared and ultraviolet behavior of Green's functions were derived which correspond to summing up contributions from all orders of perturbation theory. The results obtained modify or sometimes even contradict the approximative statements of perturbation theory. In recent years renormalization group properties have been extensively studied further [4–19]. In particular, new powerful methods were developed by Callan and Symanzik [6, 7]. Among the many important conclusions of the renormalization group approach the most remarkable one is perhaps the phenomenon of asymptotic freedom which has had an enormous impact on our understanding of elementary particle interactions [17–19].

The purpose of this and a forthcoming paper is to set up the renormalization group within the abstract approach of quantum field theory, independent of

0010-3616/80/0076/0039/$05.20