Commun. Math. Phys. 76, 1–26 (1980)

Communications in
Mathematical
Physics

Nonlinear Dynamics of the Infinite Classical Heisenberg Model: Existence Proof and Classical Limit of the Corresponding Quantum Time Evolution

Pierre A. Vuillermot*†
School of Mathematics, Georgia Institute of Technology, Atlanta, 30332 USA

Abstract. For any initial spin configuration we prove the existence, unicity and regularity properties of the solution of Hamilton's equations for the infinite classical Heisenberg model with stable interactions, along with the essential selfadjointness of the associated Liouville operator. We also prove new properties of SU(2)-coherent states which, together with the concept of Trotter approximations for one-parameter (semi-) groups, are used to show that this dynamics is nothing but the classical limit of the time evolution generated by the infinite quantum (suitably normalized) Heisenberg Hamiltonian. The classical limit emerges when the spin magnitude S goes to infinity while Plank's constant $\hbar$ goes to zero, their product $S\hbar$ remaining fixed. The main results are stated in the form of intertwining relations between the classical and the quantum unitary group.

Introduction

Rigorous results about the time evolution of systems with infinitely many degrees of freedom are rather scarce, although the last few years have revealed new contributions in the field by various authors, among whom Lanford, Lebowitz and Lieb [1] who considered a lattice system of anharmonic oscillators of arbitrary dimension, and more recently Dobrushin and Fritz [2] who considered one- and two dimensional continuous systems with singular interactions. We consider here the classical Heisenberg model with stable interactions. In section 1 we prove the existence, unicity and regularity properties of the solution of Hamilton's equations for any initial spin configuration, and essential selfadjointness of the corresponding Liouville operator. We then derive in section 2 new properties of SU(2)-coherent states which, along with the concept of Trotter approximations for one-parameter contraction (semi-) groups, are used to prove con-

* Work supported in part by the Swiss National Science Foundation under Grant 820-436-76 and in part by the U.S. Department of Energy under contract EG-77-C-03-1538.
† Also at Physical Dynamics, Inc., La Jolla, CA 92037 USA

0010-3610/80/0076/0001/$05.20