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# On the Distributions Corresponding to Bounded Operators in the Weyl Quantization

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**Abstract.** Using properties of an integral transform giving directly the matrix elements of a quantum mechanical operator from the corresponding classical function, we restrict the class of distributions corresponding to bounded operators. As a consequence, we can exhibit a class of functions yielding trace-class operators, and give a bound on their trace-norm.

## 1. Introduction

In the Weyl quantization procedure classical functions on phase space are "decomposed" by harmonic analysis in exponential functions, which are then replaced by the corresponding "elementary building blocks for operators", i.e. by the Weyl operators. One obtains in this way a map

$$f \mapsto \mathbf{Q}f \tag{1}$$

from functions on phase space to operators on a Hilbert space $\mathscr{H}$ carrying an irreducible representation of the Weyl commutation relations. It has been shown in [1] that this map can also be defined using only $f$ (skipping the harmonic analysis step) and replacing the Weyl operators by so-called Wigner operators, which are simply the Weyl operators multiplied by parity.

In both cases (the original Weyl procedure, and the prescription in [1]), the quantization formula written down is to be understood in a formal sense as long as the class of functions considered is not specified. It has been shown [2, 3] that the Weyl quantization map (1) is a unitary map from $L^2(E)$, the square integrable functions on phase space, to $\mathscr{L}_{\mathrm{HS}}(\mathscr{H})$, the space of Hilbert-Schmidt operators on $\mathscr{H}$:

$$\int dv f^*(v) g(v) = \mathrm{Tr}\,(\mathbf{Q}f^* \mathbf{Q}g)\,. \tag{2}$$

For $f$ in $L^1(E)$, the absolutely integrable functions on phase space, the operator $\mathbf{Q}f$ is compact [4], and for $f$ in the Schwarz space $\mathscr{S}(E)$, the operator $\mathbf{Q}f$ is trace-class

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