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Exponential Decay in the Stark Effect

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Abstract. Let $H = -\Delta + V + Fx_1$ with $V(x_1, x_\perp)$ analytic in the first variable and $V(x_1 + ia, x_\perp)$ bounded and decreasing to zero as $x \to \infty$ for each $a \in \mathbb{R}$. Let ψ be an eigenvector of $-\Delta + V$ with negative eigenvalue. Among our results we show that for $F \neq 0$, $(\psi, e^{-itH}\psi)$ decays exponentially at a rate governed by the positions of the resonances of H. This exponential decay is in marked contrast to "conventional" atomic resonances for which power law decay is the rule.

I. Introduction

The phenomenon of exponential decay associated with resonances is well known in quantum mechanics. Arguments which predict this phenomenon can be found in almost any elementary quantum mechanics text (see, for example, [1]). One imagines (for example) a Hamiltonian of the form $H_0 = -\Delta + V$ to be weakly perturbed by an operator W which causes an eigenvalue E_0 of H_0 to disappear into the continuum of $H = H_0 + W$. If we prepare our system at $t = 0$ in a state ψ_0 with $H_0\psi_0 = E_0\psi_0$, non-rigorous arguments indicate [1, 2] that under rather general conditions, after a very short time one has

$$(\psi_0, e^{-itH}\psi_0) \cong e^{-itE_r} \tag{1.1}$$

where $E_r = E_0 + \Delta E - i\Gamma/2$. (Here we have assumed $(\psi_0, \psi_0) = 1$.) ΔE is the energy shift due to W which can be computed approximately using Rayleigh–Schrödinger perturbation theory and Γ is the transition rate given by Fermi's Golden Rule [1].

The validity of an equation such as (1.1) has been discussed briefly by Simon [3] in the dilation-analytic framework. Simon considers Hamiltonians H which are bounded below. In this case he concludes that the best one could hope for is an approximate validity when t is not too large (nor too small). The reason for the restriction to times which are not too large is easy to understand from the following well known argument: Suppose that a bound of the form

$$|(\psi_0, e^{-itH}\psi_0)| \leqq Ce^{-\alpha|t|} \tag{1.2}$$

were true for some $\alpha > 0$ and all $t \geqq 0$ (and thus by the self-adjointness of H for all

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