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A Multi-Channel Scattering Theory for Some Time Dependent Hamiltonians, Charge Transfer Problem

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Abstract. Scattering theory for time dependent Hamiltonian $H(t) = -(1/2)\,\Delta + \sum V_j(x - q_j(t))$ is discussed. The existence, asymptotic orthogonality and the asymptotic completeness of the multi-channel wave operators are obtained under the conditions that the potentials are short range: $|V_j(x)| \leqq C_j(1+|x|)^{-2-\varepsilon}$, roughly spoken; and the trajectories $q_j(t)$ are straight lines at remote past and far future, and $|q_j(t) - q_k(t)| \to \infty$ as $t \to \pm\infty\,(j \neq k)$.

1. Introduction

The purpose of this paper is to study the scattering theory for a class of Schrödinger equations with time dependent potentials

$$i\frac{\partial u}{\partial t}(t,x) = -\frac{1}{2}\,\Delta\, u(t,x) + \sum_{j=1}^{N} V_j(x - q_j(t))u(t,x), \tag{1.1}$$

where $q_j(t) \in \mathbb{R}^n\,(n \geqq 3)$ are the functions of $t \in \mathbb{R}^1$ which are straight lines at remote past and far future.

Suppose that N-centres of forces are traveling along the given trajectories $q_j(t)\,(j = 1, 2, \ldots, N)$ each of which acts on a quantum mechanical particle of mass 1 through the potential $V_j(x)$, then the Schrödinger equation for the particle is written as (1.1). If $|q_j(t) - q_k(t)| \to \infty$ as $|t| \to \infty$ sufficiently rapidly in conjunction with the rate of decay of the potentials, one would naturally expect that the behaviour of the particle in far future or remote past are classified into $(N+1)$-ways: (1) The particle behaves like a free particle; (2) the particle travels with one of the centres $q_j(t)$ forming a bound state around the centre $(j = 1, 2, \ldots, N)$. We shall prove in this paper that this is actually what is going on with the equation (1.1) under a suitable condition. In physics literature these centres of forces are usually supposed to be atoms and ions, and the particle to be the electron. In such case the scattering theory

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