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Classical Bounds on Quantum Partition Functions*

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Abstract. Explicit bounds on the quantum partition functions are given in terms of classical partition functions, incorporating effective pair potentials, which account for Fermi- and Bose-statistics, respectively. The bounds may be used for the limit $\hbar \to 0$ and eventually for showing the interchangeability of the classical with the thermodynamic limit. A simple derivation of the thermodynamic limit for free particles with general dispersions is given.

Introduction

The correspondence principle for partition functions is quite an old problem. There exist heuristic discussions in text books and expansions in $\hbar$, but they are mathematically unsatisfactory. The problem may be divided into two points:
1) How is the trace over the Hilbert space related to the phase space integral?
2) How does the $1/N!$ for fermions and bosons originate?
The first question is settled in some detail by now [1–9], the second one is answered in this paper.

The main results appear in 2*d* and 2*e*, equation (29), (30), (43). They concern the canonical ensemble and read, expressed in the free energy:

$$F_{cl}(H_{cl}) \leqq F_F(H) \leqq F_{cl}(\hat{H}_{cl} + V_F(\hbar, \beta)) \quad \text{for fermions}$$

and

$$F_{cl}(H_{cl} - V_B(\hbar, \beta)) \leqq F_B(H) \leqq F_{cl}(\hat{H}_{cl}) \quad \text{for bosons}$$

(cl stands for "classical", F for fermions, B for bosons, $\beta = 1/kT$). The Hamiltonians are supposed to be of the form

$$H = \sum_{i=1}^{N} p_i^2 + V(x_1 \dots x_N)$$

and H_{cl} is the same function of canonical coordinates instead of the operators p_i, x_i. Almost no condition restricts the set of allowed potentials V, one has only to make

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