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On a Characterization of the State Space of Quantum Mechanics

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Abstract. A characterization of state spaces of Jordan algebras by Alfsen and Shultz is improved to a form with more physical appeal (proposed by Wittstock) in the simplified case of a finite dimension.

1. Introduction

In recent years there have been a number of works on the characterization of state spaces of W^* and C^* algebras [1–3]. We present here another version, which seems to have a somewhat better physical appeal, though applicable only to a special situation (finite type I cases in the sequel).

Our axioms are very close to those of Alfsen and Shultz ([4], Sect. 6) for type I Jordan algebras except that we replace their P-projections by a weaker notion of filtering projections, which has been suggested by Wittstock [5].

The state space is assumed to be a compact convex set K as usual, where the convex combination of points in K represents a mixture of physical states, extremal points of K corresponding to pure states. We make the simplifying assumption that *the dimension of K is finite.*

The first axiom is in terms of *filtering projections*, which have a physical interpretation of the measuring process of the first kind for questions (observables with yes or no answers), as will be described in Definition 2.2. *Axiom 0* in Sect. 3 requires the existence of sufficiently many filtering projections (i.e. one for each face of K) satisfying a certain consistency condition with an obvious physical interpretation.

The filtering projection P_φ associated with a pure state φ defines a number $\langle \varphi, \psi \rangle$ for another pure state ψ through the relation

$$P_\varphi \psi = \langle \varphi, \psi \rangle \varphi , \tag{1.1}$$

with an interpretation of the transition probability. Our second Axiom is

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