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The Lipatov Argument

Thomas Spencer*

Department of Mathematics, Rutgers University, New Brunswick, NJ 08903, USA

Abstract. Lipatov's argument gives a formula for evaluating asymptotically the large order perturbation coefficients for the anharmonic oscillator or (ϕ^4) quantum field models. We give a partial justification of the argument which enables us to prove that the radius of convergence of the Borel transform of the pressure for lattice ϕ^4 models given by

$$\exp\left[\inf_{\phi}\left\{\tfrac{1}{2}\sum_{j}[(\nabla\phi)^2(j)+\phi(j)^2]-\log\sum\phi(j)^4\right\}-2\right].$$

Let $E(\lambda)$ be the ground state energy for the anharmonic oscillator

$$H(\lambda)=-\tfrac{1}{2}\frac{d^2}{dx^2}+\frac{x^2}{2}+\lambda x^4-\tfrac{1}{2}.$$

It is well known that $E(\lambda)$ has an asymptotic but divergent series in λ

$$E(\lambda)\approx\sum_{n=0}^{\infty}a_n\lambda^n. \tag{1}$$

We shall discuss the behavior of a_n for large n.

In 1973 Bender and Wu [2] developed W.K.B. techniques to obtain asymptotics of the form

$$a_n\approx C_0C_1^n n^{\alpha} n!\left(1+\frac{O(1)}{n}\right) \tag{2}$$

with explicit expressions for C_0, C_1, and α. Recently Benassi et al. [1] have rigorously established (2) along the lines of Bender and Wu. Several years later Lipatov [5] developed steepest descent methods for functional integrals which he and Brezin et al. [3] applied to quantum field models to obtain results analogous

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