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Remarks on the Global Markov Property

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Abstract. The global Markov property is established for the + state and the − state of attractive lattice systems (e.g., the ferromagnetic Ising model and most other systems for which the FKG inequalities are satisfied) and of the (continuum) Widom Rowlinson model.

1. Introduction

Consider the Gibbs states for a *nearest neighbor* interaction Φ on the lattice $\Gamma=\mathbb{Z}^d$ with finite state space S. These are the probability measures μ on $\Omega=S^\Gamma$ whose conditional probabilities satisfy the DLR equations [1, 2]:

$$\mu\{\sigma_\Lambda=y|x_{\Gamma-\Lambda}\}=\frac{e^{-h_\Lambda^\Phi(y,x)}}{\sum\limits_{y'\in\Omega_\Lambda} e^{-h_\Lambda^\Phi(y',x)}}. \tag{1.1}$$

Here Λ is a finite subset of Γ, $\Omega_\Lambda=S^\Lambda$ is the set of configurations on Λ, $y\in\Omega_\Lambda$, $x\in\Omega$, $x_{\Gamma-\Lambda}$ is the configuration in $\Omega_{\Gamma-\Lambda}$ obtained by restricting x to $\Gamma-\Lambda$, $\sigma_\Lambda(x)=x_\Lambda$, and $h_\Lambda^\Phi(y,x)$ is the energy in Λ produced by the interaction (Φ) of the spins in Λ (described by y) with themselves and with the spins outside of Λ (described by $x_{\Gamma-\Lambda}$). In particular the state μ satisfies the local Markov property: For any bounded $f\in\mathscr{F}_F$,

$$E(f|\mathscr{F}_{\Gamma-F})=E(f|\mathscr{F}_{\partial F}), \tag{1.2}$$

where $F=\Lambda$ is a finite subset, ∂F s the set of sites in $\Gamma-F$ which are nearest neighbors to at least one site in F, $\mathscr{F}_D$ is the sub-σ-algebra generated by σ_D, $D\subset\Gamma$, $E(\cdot|\mathscr{F}_D)$ denotes the conditional expectation given $\mathscr{F}_D$, and $f\in\mathscr{F}_F$ means f is bounded and $\mathscr{F}_F$ measurable. We write $\mathscr{F}$ for $\mathscr{F}_\Gamma$.

Markov chains by definition satisfy (1.2) ($d=1$) for $F=\{n,n+1,n+2,\dots\}$ and, in fact, satisfy (1.2) for any F whatsoever. A state μ satisfying (1.2) for any (not

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