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Intermittent Transition to Turbulence in Dissipative Dynamical Systems

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Abstract. We study some simple dissipative dynamical systems exhibiting a transition from a stable periodic behavior to a chaotic one. At that transition, the inverse coherence time grows continuously from zero due to the random occurrence of widely separated bursts in the time record.

Introduction

A number of investigators [1] have observed in convective fluids an intermittent transition to turbulence. In these experiments the external control parameter, say r, is the vertical temperature difference across a Rayleigh-Bénard cell. Below a critical value r_T of this parameter, measurements show well behaved and regular periodic oscillations. As r becomes slightly larger than r_T the fluctuations remain apparently periodic during long time intervals (which we shall call "laminar phases") but this regular behavior seems to be randomly and abruptly disrupted by a "burst" on the time record. This "burst" has a finite duration, it stops and a new laminar phase starts and so on. Close to r_T, the time lag between two bursts is seemingly at random and much larger than – and not correlated to – the period of the underlying oscillations. As r increases more and more beyond r_T it becomes more and more difficult and finally quite impossible to recognize the regular oscillations (see Fig. 1).

This sort of transition to turbulence is also present in simple dissipative dynamical systems [2] such as the Lorenz model [2a]. We present here the results of some numerical experiments on this problem.

When a burst starts at the end of a laminar phase this denotes an instability of the periodic motion due to the fact that the modulus of at least one Floquet multiplier [3] is larger than one. This may occur in three different ways: a real Floquet multiplier crosses the unit circle at $(+1)$ or at (-1) or two complex conjugate multipliers cross simultaneously. To each of these three typical crossings we may associate one type of intermittency that we shall call for convenience type

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