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Normal Fluctuations and the FKG Inequalities

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Abstract. In a translation invariant pure phase of a ferromagnet, finite susceptibility and the FKG inequalities together imply convergence of the block spin scaling limit to the infinite temperature Gaussian fixed point. This result is presented in a rather general probabilistic context and is applicable to infinite cluster density fluctuations in percolation models and to boson field fluctuations in (Euclidean) Yukawa quantum field theory models as well as to magnetization fluctuations in Ising models.

1. Introduction and Results

We consider a d-dimensional cubic lattice of random variables, $\{X_k : k=(k_1, \ldots, k_d) \in \mathbb{Z}^d\}$, and for each integer $n \geqq 1$ the associated block variables $\{X_k^n : k \in \mathbb{Z}^d\}$ defined by

$$X_k^n = (S_k^n - \langle S_k^n \rangle)/n^{d/2}, \tag{1}$$

where $\langle \cdot \rangle$ denotes expectation,

$$S_k^n = \sum_{j \in B_k^n} X_j, \tag{2}$$

and B_k^n is a "block" of side length n located near nk:

$$\begin{aligned} B_k^n &= \{j : nk_l \leqq j_l < n(k_l+1) \quad \text{for} \quad l=1, \ldots, d\} \\ &= nk + B_0^n. \end{aligned} \tag{3}$$

We will present a simple natural set of conditions on $\{X_k\}$ (see Theorem 2 below) which insure (in renormalization group terminology [26]) convergence of $\{X_k^n\}$ as $n \to \infty$ to the infinite temperature Gaussian fixed point; i.e., which insure that as $n \to \infty$,

$$\{X_k^n : k \in \mathbb{Z}^d\} \to \{Z_k : k \in \mathbb{Z}^d\}, \tag{4}$$

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