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Representation Theory and Integration of Nonlinear Spherically Symmetric Equations to Gauge Theories

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Abstract. A constructive proof of complete integrability of spherically symmetric self-dual equations in Euclidean space R_4 for an arbitrary embedding of SU(2) in an arbitrary gauge group G is given on the base of Lax-type representation and representation theory. The equations are solved explicitly for the case of simple Lie groups G.

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The aim of our paper is to give a constructive proof of complete integrability of spherically symmetric classical self-dual equations in R_4 for an *arbitrary* embedding of SU(2) in an *arbitrary* gauge group G of rank n. The technique for dealing with this essentially nonlinear two-dimensional partial differential system involves Lax-type representation [1] (see also [2]) and representation theory, which enable us to reformulate the integrability problem in terms of the main notions of the representation theory of the corresponding group and to solve the equations explicitly. In this the spherically symmetric instantons in R_4 and monopoles in Minkowski space $R_{3,1}$ (with Higgs Scalar field in adjoint representation of G) in a classical sense form a special subset of our solutions under the relevant boundary conditions. All our previous results [3] concerning the construction of exact solutions to the spherically symmetric self-dual equations for the minimal embedding of SU(2) in G with $\prod_1^n \otimes U(1)$ invariance subgroup are consequences of the general scheme of the present paper.

Note also that in static case the corresponding system of equations for the minimal embedding of SU(2) in G describes generalized (finite, nonperiodic) Toda lattice (see e.g. [4]). In the framework of our consideration [5] the solution to this system as well as its two-dimensional generalization arise as a particular case.