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The Critical Probability of Bond Percolation on the Square Lattice Equals 1/2*

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Abstract. We prove the statement in the title of the paper.

1. Introduction

Broadbent and Hammersley [2], introduced the following percolation problem. Let $\mathscr{L}$ be the graph in the plane whose vertices are the integral vectors (i.e., elements of $\mathbb{Z}^2$) and whose edges or bonds are the segments connecting two adjacent vertices (we call two vertices v' and v'' of $\mathscr{L}$ adjacent if the distance between them equals 1). Let each bond of $\mathscr{L}$ be open or passable with probability p, and closed or blocked with probability $q=1-p$, and assume that open- or closedness for all different bonds is chosen independently. The percolation probability is defined as

$$\theta(p)=P\{\text{the origin is part of an infinite connected open set in } \mathscr{L}\}, \tag{1.1}$$

and the critical probability p_H as

$$p_H=\inf\{p:\theta(p)>0\}. \tag{1.2}$$

Hammersley [5], [6] proved

$$\frac{1}{\lambda} \leqq p_H \leqq 1-\frac{1}{\lambda}. \tag{1.3}$$

where λ is the socalled connectivity constant of $\mathscr{L}(\lambda \approx 2.639$, see [9]). Harris [7] improved the lower bound to

$$p_H \geqq \tfrac{1}{2}. \tag{1.4}$$

Various results and numerical evidence (see [17], or [15] Chap. III, for a brief summary) indicated that $p_H=\frac{1}{2}$, and most people seem to have accepted the truth

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