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On the Integrability of Classical Spinor Models in Two-Dimensional Space-Time

V. E. Zakharov and A. V. Mikhailov

L. D. Landau Institute of Theoretical Physics, Academy of Sciences of USSR, Moscow V-334, USSR

Abstract. Well known classical spinor relativistic-invariant two-dimensional field theory models, including the Gross-Neveu, Vaks-Larkin-Nambu-Jona-Lasinio and some other models, are shown to be integrable by means of the inverse scattering problem method. These models are shown to be naturally connected with the principal chiral fields on the symplectic, unitary and orthogonal Lie groups. The respective technique for construction of the soliton-like solutions is developed.

Introduction

Classical spinor systems (classical analogs of fermion fields with c-number values) have often been considered in the physical literature. First of all, there are the models of Nambu and Jona-Lasinio [1] and Vaks and Larkin [2]:

$$\begin{aligned} \partial_\eta \varphi^\alpha &= i/2\psi^\alpha \sum_\beta \psi^{*\beta}\varphi^\beta \\ \partial_\xi \psi^\alpha &= i/2\varphi^\alpha \sum_\beta \varphi^{*\beta}\psi^\beta \end{aligned} \tag{1}$$

and the Gross-Neveu model [3]:

$$\begin{aligned} \partial_\eta \varphi^\alpha &= -i\psi^\alpha \sum_\beta (\psi^{*\beta}\varphi^\beta + \varphi^{*\beta}\psi^\beta) \\ \partial_\xi \psi^\alpha &= -i\varphi^\alpha \sum_\beta (\varphi^{*\beta}\psi^\beta + \varphi^\beta\psi^{*\beta}), \end{aligned} \tag{2}$$

where $\eta = t + x$, $\xi = t - x$.

Models (1), (2) are relativistically invariant and represent systems of N massless Dirac equations in a two-dimensional space-time with nonlinear (cubic) terms. Models (1), (2) correspond to the actions

$$S = \int dt dx \left[\sum_\alpha (i\varphi^{*\alpha}\partial_\eta \varphi^\alpha + i\psi^{*\alpha}\partial_\xi \psi^\alpha) - \tfrac{1}{2}\left| \sum_\alpha \psi^{*\alpha}\varphi^\alpha \right|^2 \right], \tag{3}$$

$$S = \int dt dx \left[\sum_\alpha (i\varphi^{*\alpha}\partial_\eta \varphi^\alpha + i\psi^{*\alpha}\partial_\xi \psi^\alpha) - \tfrac{1}{2}\left(\sum_\alpha (\psi^{*\alpha}\varphi^\alpha + \psi^\alpha\varphi^{*\alpha}) \right)^2 \right]. \tag{4}$$