Two Remarks on the Computer Study
of Differentiable Dynamical Systems

Massimo Campanino*

Institut des Hautes Etudes Scientifiques, 35, Route de Chartres, F-91440 Bures-sur-Yvette, France

Abstract. In the first part of this note we find conditions under which the frequency spectrum of a transformation exhibits delta functions. In the second part we show that if an ergodic flow on an m-dimensional manifold has $m-1$ strictly negative characteristic exponents, then the measure is concentrated either on a fixed point or on a closed attracting orbit.

1. Introduction

In this note we examine some relations between the properties of an invariant measure (for a transformation or a flow) and some of its numerical characteristics, viz. the frequency spectrum and the characteristic exponents. Our remarks are relevant to studies by computer.

In Sect. 2 we find conditions under which the frequency spectrum of a transformation exhibits delta functions. A situation like the one we describe (cyclic permutation of attractors) arises in the case of the Hénon map for various values of the parameter (see [3] and [2]). In Sect. 3 we study an ergodic flow on a compact manifold. We show that if all the characteristic exponents but one are strictly negative, then all the measure is carried by a fixed point or by an attracting closed orbit. This generalizes the analogous result for transformations ([5], Corollary 6.2). For the basic definitions and results on transformations and flows see [1] and [4].

2. Presence of Delta Functions in the Frequency Spectrum

We consider a continuous transformation T of a topological space. We assume that there is a finite number of disjoint compact sets which are cyclically permuted by T and a Borel T invariant measure, whose support is the union of the compacts. A standard method for studying the dynamical system generated by a transfor-

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* Permanent address: Istituto Matematico "G. Castelnuovo", Universita degli Studi di Roma, Piazzale delle scienze, I-00100 Roma, Italy