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On the Abundance of Aperiodic Behaviour for Maps on the Interval

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Introduction

In this paper, we study the following problem: Given a one-parameter family of continuous maps of the interval [0, 1] into itself, how many of these maps show aperiodic behaviour? For a particular family of maps *containing a quadratic part* we are able to show that for many values of the parameter (in fact for a set of positive Lebesgue measure) these maps do present aperiodic behaviour.

The parameter in question will be called δ (and is always supposed to be small, positive) and the particular family of functions is defined by

$$f_\delta(x) = \begin{cases} 2x & \text{if } 0 \leqq x \leqq \frac{1}{2} - \delta \\ 2(1-x) & \text{if } \frac{1}{2} + \delta \leqq x \leqq 1 \\ 1 - \delta - (x - \frac{1}{2})^2/\delta & \text{if } x \in E_\delta, \end{cases}$$

where $E_\delta = \{x \mid |x - \frac{1}{2}| < \delta\}$, so the graph of f_δ is

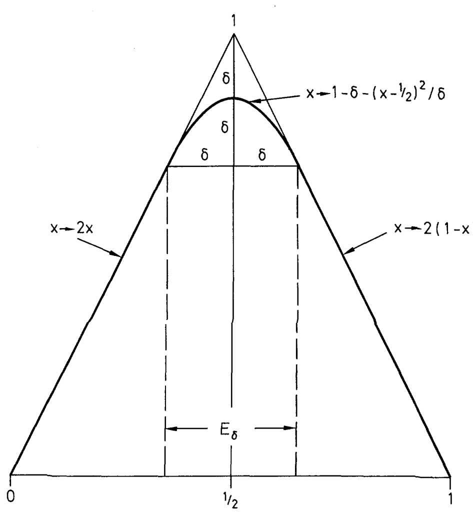

Fig. 1. The function f_δ for $\delta = 0.15$

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