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Note

The Operators Governing Quantum Fluctuations of Yang-Mills Multi-Instantons on S^4 and Their Seeley Coefficients

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Abstract. We give explicit expressions for the Seeley coefficients of the fluctuation operator and the operator that appears in the Faddeev-Popov determinant, which arise in the calculation of quantum fluctuations around Yang-Mills multi-instantons.

In the calculation of quantum fluctuations around multi-instanton configurations it is of interest to know the Seeley coefficients for the fluctuation, and the gauge fixing operators [1]. In this note we shall give explicit expressions for these coefficients.

We work on S^4, the one-point compactification of $\mathbb{R}^4$. Let $\square$ be a second order, self-adjoint, non-negative elliptic operator on S^4. Then it is well known [2] that the series

$$h_t(\square) = \sum_\lambda e^{-t\lambda}$$

converges for any $t>0$. The summation extends over all eigenvalues, λ, of $\square$ with the appropriate multiplicities. Furthermore, $h_t(\square)$ has an asymptotic expansion

$$h_t(\square) \equiv \mathrm{Tr}\, e^{-t\square} \sim t^{-2}\psi_2(\square) + t^{-1}\psi_1(\square) + \psi_0(\square) + O(t^\delta), \delta > 0$$

for $t \downarrow 0$. The $\psi_k(\square)$'s are known as the Seeley coefficients of $\square$. Moreover each $\psi_k(\square)$ can be expressed as an integral over S^4 of a certain measure $\psi_k(x|\square)$dvol. $\psi_k(x|\square)$ depends polynomially on the coefficients of $\square$ and their derivatives. They can be expressed in terms of curvature invariants. In fact, the above asymptotic expansion is a consequence of a local expansion. Indeed, if $K_t(x,y)$ is the kernel of the operator $e^{-t\square}$ then

$$K_t(x,x) \sim t^{-2}\psi_2(x|\square) + t^{-1}\psi_1(x|\square) + \psi_0(x|\square) + O(t^\delta).$$

From this it also follows that

$$\hat{\psi}_k(x|\lambda\square) = \lambda^{-k}\psi_k(x|\square), \quad \lambda \in R^+, \quad k = 0, 1, 2.$$

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