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Boundedness of Total Cross-Sections in Potential Scattering. II

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Abstract. If, in addition to the condition

$$\frac{1}{(4\pi)^2}\int d^3x d^3x' \frac{|V(x)|\,|V(x')|}{|x-x'|^2} < 1$$

in units where $2M/\hbar^2 = 1$, which guarantees that the total cross-section averaged over incident directions is finite, we have also

$$\frac{1}{(4\pi)}\int d^3x d^3x' \frac{|V(x)|\,|V(x')|}{|x-x'|}$$

finite, the total cross-section is finite for all energies and all directions of the incident beam.

We have recently shown, in a paper bearing the same title [1], that if the quantity

$$I = \frac{1}{(4\pi)^2}\int d^3x d^3x' \frac{|V(x)|\,|V(x')|}{|x-x'|^2} \tag{1}$$

is such that

$$I < 1 \tag{2}$$

in units where $2M/\hbar^2 = 1$, M being the mass of the particle scattered by the potential V, the total cross-section averaged over angles

$$\bar{\sigma}_T(k) = \int \frac{d\Omega_k}{4\pi}\sigma_T(\mathbf{k}) \tag{3}$$

is finite. More precisely

$$\bar{\sigma}_T(k) < \frac{4\pi}{k^2}\frac{I}{(1-\sqrt{I})^2}. \tag{4}$$