

A Characterization of Markovian Homogeneous Multicomponent Gaussian Fields

G. O. S. Ekhaguere

Department of Mathematics, University of Ibadan, Ibadan, Nigeria

Abstract. Necessary and sufficient conditions are given for a certain class of homogeneous multicomponent Gaussian generalized stochastic fields to possess a Markov property equivalent to Nelson's. The class of Markov fields so characterized has as a subclass the class of Markov fields which lead by Nelson's Reconstruction Theorem to some covariant (free) quantum fields.

Introduction

In 1948, Levy [1] defined a Brownian motion indexed by the d -dimensional Euclidean space \mathbb{R}^d . With this Brownian motion in mind, he introduced [2] the notion of *Markov property of finite order* for (ordinary) stochastic fields, and conjectured [2] that the Brownian motion indexed by $\mathbb{R}^{2\nu+1}$, $\nu = a$ nonnegative integer, is Markovian of order $\nu+1$. It was left to McKean [3] to establish the veracity of Levy's conjecture. Recently, Molchan [4] has furnished an alternative proof of Levy's conjecture while Pitt [5] has extended McKean's proof to arbitrary Gaussian stochastic fields which are Markovian of some finite order. In his proof, Molchan employs aspects of the theory of elliptic partial differential equations and the notion of the reproducing kernel Hilbert space [6] associated with a stochastic field. By a blending of the preceding ideas with those of McKean [3] and by applying Peetre's characterization [7] of differential operators, Pitt characterized a finite order Markovian Gaussian stochastic field, under some assumptions, by identifying the inner product of its reproducing kernel Hilbert space with the Dirichlet form [8] of a strongly elliptic partial differential operator.

Results, of the above type are, of course, certainly of relevance at the initial stages of development of a theory of Markov stochastic fields. But stochastic fields which are Markovian of finite order are clearly only a special class of Markovian fields. Indeed, McKean employs a generalization of Levy's notion of Markov property in [3] and his extended definition thus accommodates Markov stochastic fields which are not necessarily of finite order. In [9] and [10], necessary and sufficient conditions for a homogeneous scalar Gaussian stochastic field to be