Commun. Math. Phys. 73, 43–62 (1980)

Communications in
Mathematical
Physics

Spectral and Scattering Theory for a Class of Strongly Oscillating Potentials

Monique Combescure

Laboratoire de Physique Théorique et Hautes Energies*,
Université de Paris-Sud, F-91405 Orsay, France

Abstract. Following a recent investigation by Pearson [23] on scattering theory for some class of oscillating potentials, we consider the Schrödinger operator in $L^2(\mathbb{R}^n)$ given by: $H = -e^{-U}\nabla \cdot e^{2U}\nabla e^{-U} + e^{-2U}(F + (\nabla \cdot Q))$. Here U and F are real functions of x, and Q is a $\mathbb{R}^n$-valued function of x, such that:

(1) U is bounded, and the local singularities of F and Q^2 are controlled in a suitable sense by the kinetic energy,

(2) U, Q, and F tend to zero at infinity faster than $|x|^{-1}$. We define H by a method of quadratic forms and derive the usual results of scattering theory, namely: the absolutely continuous spectrum is $[0, \infty)$ and the singular continuous spectrum is empty, the wave operators exist and are asymptotically complete. This enlarges the class of already studied strongly oscillating potentials that give rise to the scattering and spectral properties mentioned above.

1. Introduction

The spectral and scattering theory of the Schrödinger operator $H = H_0 + V$ where $H_0 = -\Delta$, and V is a real-valued function of x ($x \in \mathbb{R}^n$), has been shown to extend to potentials of the form $V = (\nabla \cdot W)$ where W is a $\mathbb{R}^n$-valued function of x, and where all assumptions on local behaviour and decrease at infinity are made on W^2 instead of $|V|$ [2, 8, 20, 28, 31]. One of the main interests of this result is that, while W^2 is chosen so as to behave like a good short-range potential, $V = (\nabla \cdot W)$ can be wildly oscillating either locally or at infinity $\left(\text{for example } V = \frac{1}{r(1+r^2)^{1+\varepsilon}} e^{1/r} \cdot \cos e^{1/r}, \varepsilon > 0\right)$ in such a way that important cancellations occur between its positive and negative parts, so that asymptotic completeness between $H_0 + V$ and H_0 holds. On the contrary, in the example found by Pearson [21] of a very

* Laboratoire associé au Centre National de la Recherche Scientifique

0010-3616/80/0073/0043/$04.00