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Arbitrary N-Vortex Solutions to the First Order Ginzburg-Landau Equations*

Clifford Henry Taubes

Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA

Abstract. We prove that a set of N not necessarily distinct points in the plane determine a unique, real analytic solution to the first order Ginzburg-Landau equations with vortex number N. This solution has the property that the Higgs field vanishes only at the points in the set and the order of vanishing at a given point is determined by the multiplicity of that point in the set. We prove further that these are the only C^∞ solutions to the first order Ginzburg-Landau equations.

1. Introduction

A mathematical model of superconductors is given by the Ginzburg-Landau equations [1]. These equations exhibit vortex solutions which may be viewed as finite energy solutions to the equations describing the two dimensional Abelian Higgs model [2]. After suitable rescaling, the equations have one coupling constant, λ, whose value determines whether the equations describe a type I or type II superconductor. The value $\lambda = 1$ is the critical value. In this case, the energy of a configuration is bounded below by a topological invariant, a multiple of the vortex number. Any configuration which achieves this minimum energy will be a solution of the Ginzburg-Landau equations. To find solutions with this minimum bound, one need only solve a set of first order coupled equations for the vector potential and the Higgs field rather than the more general second order equations. DeVega and Schnaposnik [3] in an analysis of the equations, gave a numerical argument for the existence of a cylindrically symmetric solution to the first order equations with vortex number N which has been interpreted as N-vortices superimposed at a point. Weinberg [4] recently proved that if a solution of the Ginzburg-Landau equations with vortex number N exists then the dimension of the space of moduli of this solution must be $2N$. This led him to conjecture that there exists a $2N$ parameter family of solutions with vortex number N and that the parameters of a solution may be interpreted as being the positions in $\mathbb{R}^2$ of the N vortices. Recent

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