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Renormalized G-Convolution of N-Point Functions in Quantum Field Theory: Convergence in the Euclidean Case II

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Abstract. The notion of Feynman amplitude associated with a graph G in perturbative quantum field theory admits a generalized version in which each vertex v of G is associated with a *general* (non-perturbative) n_v-point function H^{n_v}, n_v denoting the number of lines which are incident to v in G. In the case where no ultraviolet divergence occurs, this has been performed directly in complex momentum space through Bros–Lassalle's G-convolution procedure.

Note for the Reader

The general introduction of this work and the necessary mathematical material have been published in Commun. Math. Phys. Vol. 72, pp. 175–205. We now present our result on the convergence of renormalized G-convolution. In Sect. 2, a definition of our generalized renormalized integrand R_G is given: this definition closely follows Zimmermann's algorithm [7] and involves a sum of counterterms which are associated with all the G-forests: a G-forest is a subset of "non-overlapping" subgraphs of G.

In Sect. 3, we introduce the notion of "complete forest with respect to a nested set of subspaces of $E^{rm}_{(k)}$" (this is also an extension of a notion defined in [7]). This notion allows to write new expressions of R_G which are used in the following Sect. 4. The latter contains the proof of our main theorem: R_G satisfies Weinberg's convergence criterion, and thus the renormalized integral $H_G^{(\mathrm{ren})}(K)$ is a well-defined function in the Euclidean region.

2. A Generalization of Zimmermann's Renormalized Integrand

2.1. The Unrenormalized Integrand I_G

Let us consider a general connected graph G with n external lines and m independent loops. Let $\mathscr{L}$ denote the set of internal lines of G, $\mathscr{N}$ the set of its vertices, X the set of its external lines: $|X| = n$. Each internal line is considered as oriented;