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Renormalized G-Convolution of N-Point Functions in Quantum Field Theory: Convergence in the Euclidean Case I

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Abstract. The notion of Feynman amplitude associated with a graph G in perturbative quantum field theory admits a generalized version in which each vertex v of G is associated with a *general* (non-perturbative) n_v-point function H^{n_v}, n_v denoting the number of lines which are incident to v in G. In the case where no ultraviolet divergence occurs, this has been performed directly in complex momentum space through Bros–Lassalle's G-convolution procedure.

In the present work we propose a generalization of G-convolution which includes the case when the functions H^{n_v} are *not* integrable at infinity but belong to a suitable class of slowly increasing functions. A "finite part" of the G-convolution integral is then defined through an algorithm which closely follows Zimmermann's renormalization scheme. In this work, we only treat the case of "Euclidean" r-momentum configurations.

The first part which is presented here contains together with a general introduction, the necessary mathematical material of this work, i.e., Sect. 1 and appendices A and B.

The second part, which will be published in a further issue, will contain the Sects. 2, 3 and 4 which are devoted to the statement and to the proof of the main result, i.e., the convergence of the renormalized G-convolution product.

The table of references will be given in both parts.

Introduction

It has become commonly accepted in Particle Physics that various collision mechanisms may be conveniently described in the language of generalized Feynman amplitudes. These quantities are associated with "fat" Feynman graphs, in which the orthodox point-wise vertices of perturbation theory are replaced by "bubbles", whenever strong interaction processes have to be taken into account (For an up-to-date example of such description, see the well known deep-inelastic scheme [1]).

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