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On Edwards' Model for Long Polymer Chains

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Abstract. An existence theorem is proved for a probability measure on continuous paths in space, proposed by Edwards as a stochastic model for the geometric properties of long polymer chains.

1. Introduction

The problem of setting up and analysing a probabilistic model for long polymer chains which takes into account the so-called "excluded volume" effect is an old one, being described already in Kac's classic survey of probabilistic methods in physics [1]. A simple discrete model is obtained by considering the "self-avoiding" random walks on a lattice, so that the problem is to determine the asymptotic behaviour of very long walks of this kind. Since the self-avoiding random walk is not a Markov process, progress has been slow indeed. Thus the survey article of Domb [2] (1969) lists no further rigorous results beyond those established by Hammersley and Kesten (by 1964)[1]. The problem has been studied by computer with results described in detail in the cited article of Domb. We do not wish to review these results here but only to call attention to Domb's conclusion that it is possible to distinguish between long and short range properties of the polymer chain, the long range properties being sensibly independent of the detail of the interaction between the links of the chain. Thus just as the asymptotics of random walks (under rather general conditions on the distribution of the individual steps) is substantially equivalent to the study of Brownian motion (the Wiener process), the long range properties of polymer chains should be studied in an appropriate continuum model.

Such a model has been proposed by Edwards [3]. In this model the chains are represented by continuous paths $\mathbf{x}(\sigma)$, $0 \leqq \sigma \leqq 1$, in $\mathbb{R}^3$, with $\mathbf{x}(o) = \mathbf{0}$, the probability measure ν on the space of paths being given in terms of Wiener measure μ by

$$\frac{d\nu}{d\mu} = \mathscr{Z}^{-1} \exp[-gJ], \tag{1}$$

1 See also [2a] for a more recent review

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